Development of a Bayesian inference framework for near-range source term estimation and uncertainty quantification

J.P.K.W. Frankemölle, J. Camps, P. De Meutter & J. Meyers

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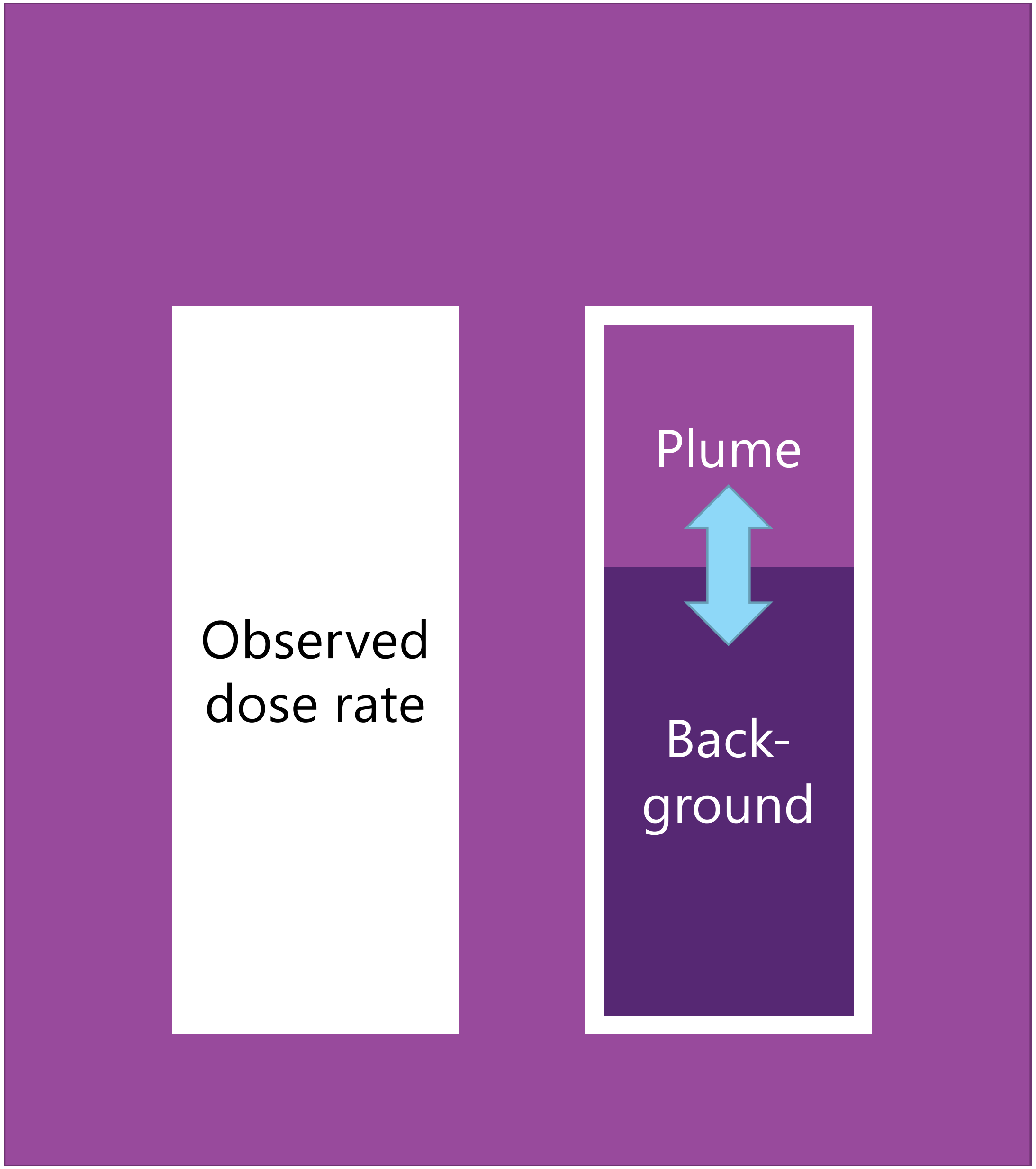
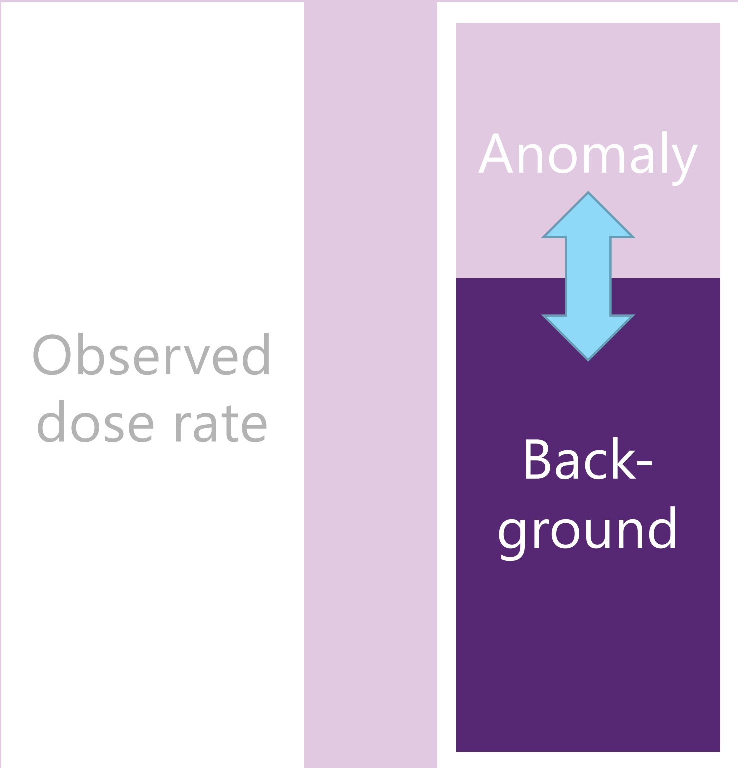
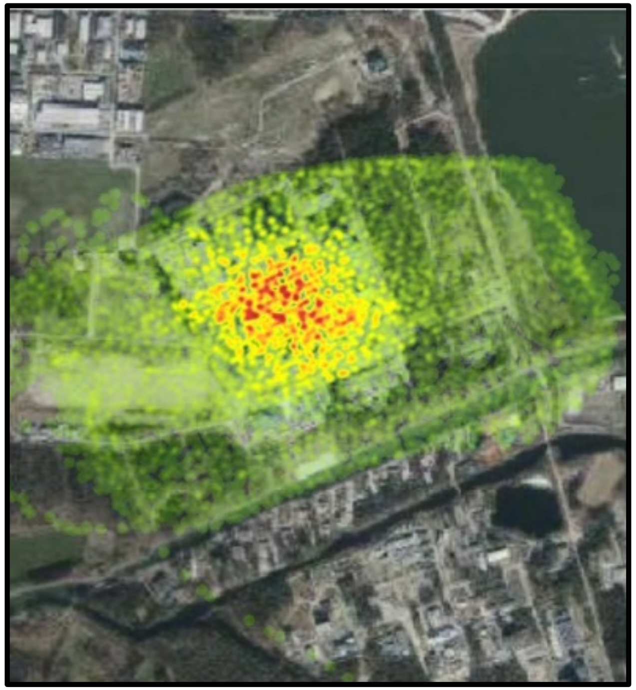


Inferring the ambient dose equivalent rate

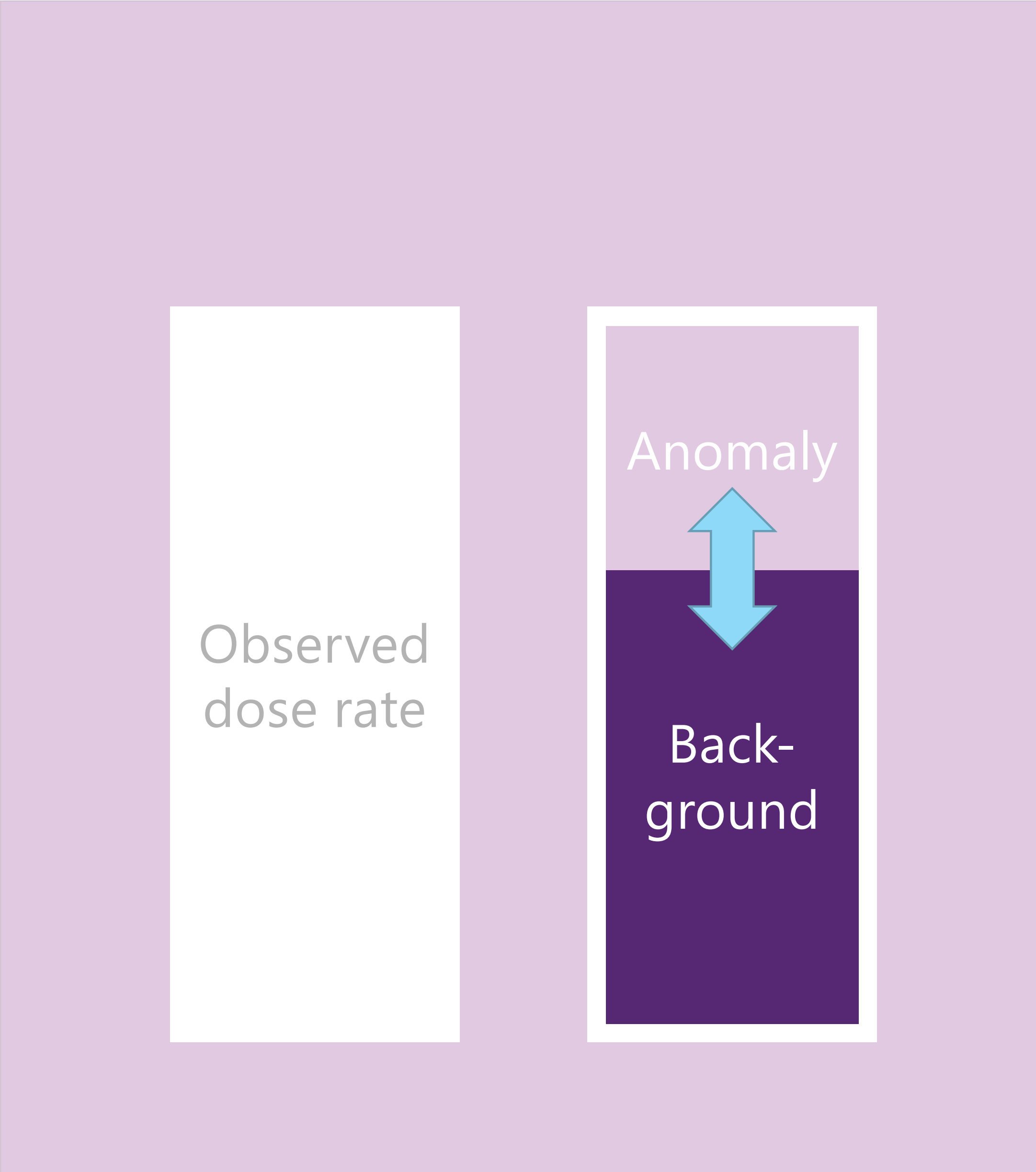
- Goal is to **calibrate the dispersion model** using dose rates through Bayesian inference
- Prerequisites
  - Good estimate of the background dose rate
  - Physics-informed uncertainty parametrisations for both background and dispersion



Background prediction



Camps, J. , Fiengo Perez, F., Geelen, S., Frankemölle, J.P.K.W. & the BUDDAWAK team (2023) 'Overview of project results, including a demonstration of the payload', *BUDDAWAK Final Workshop, 30 May 2023, Dessel.*



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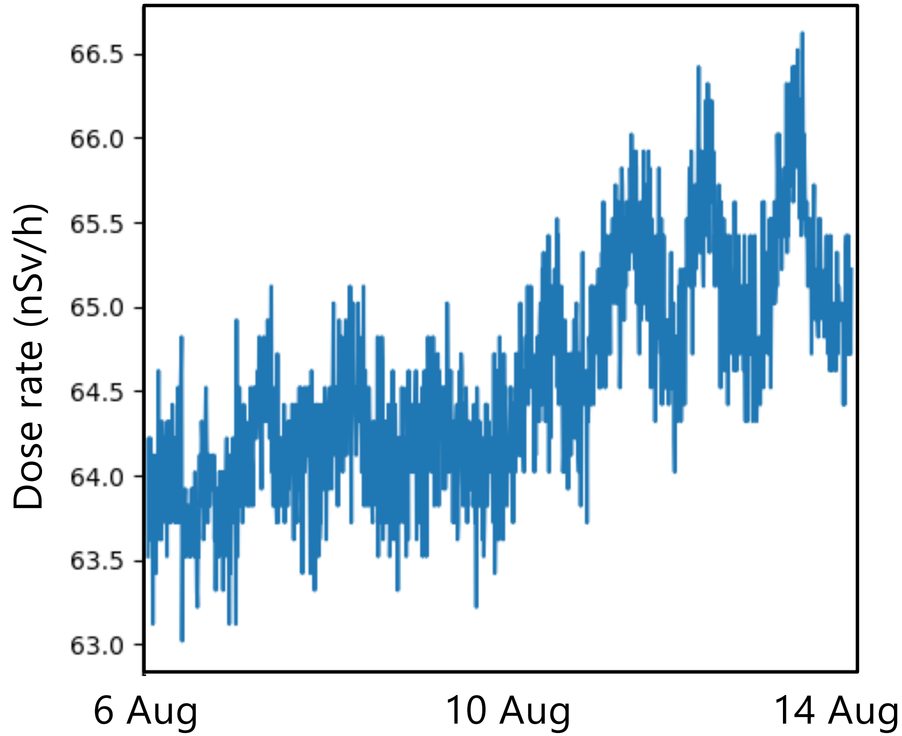

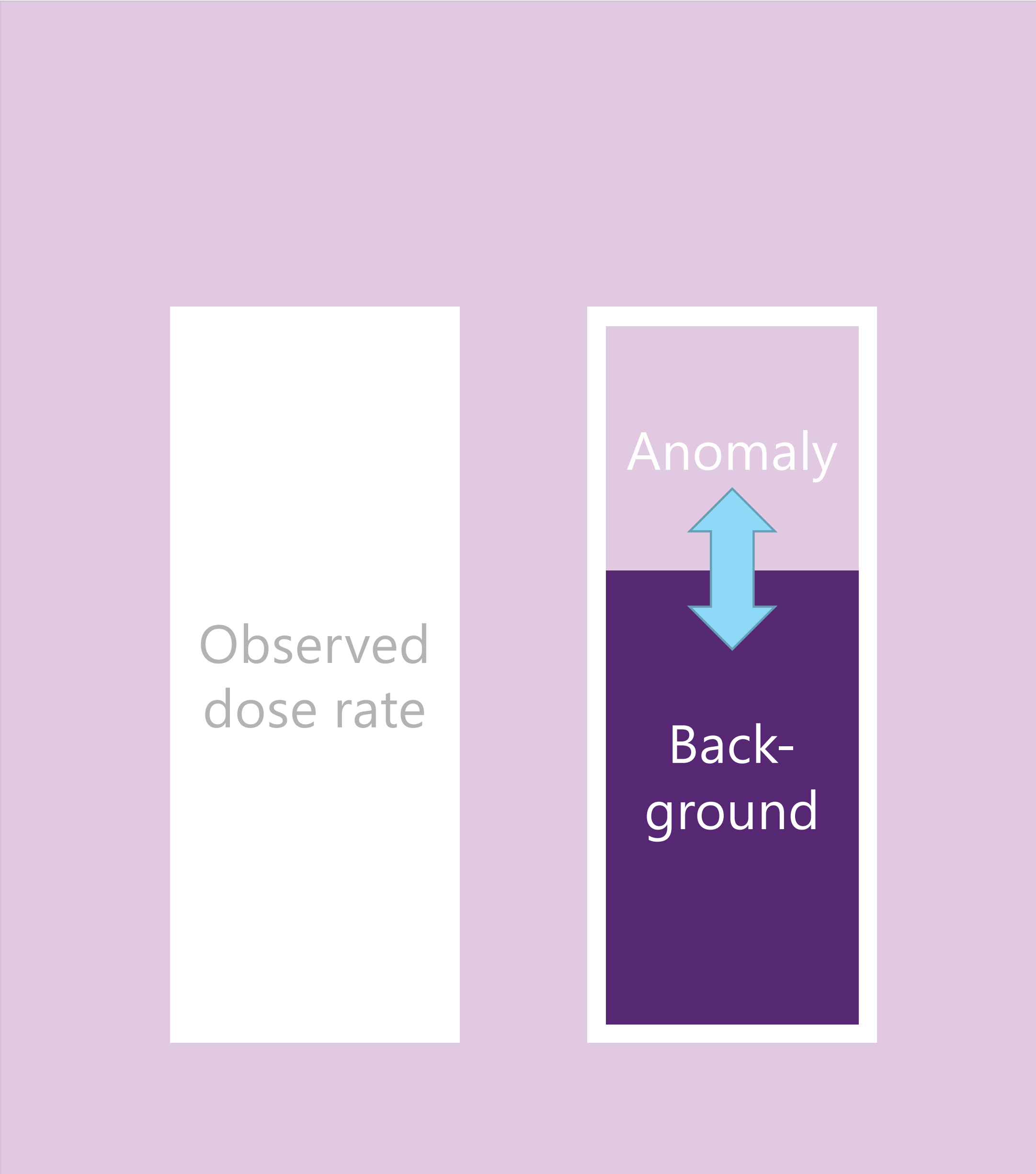

Background prediction

- Important for
  - Source reconstruction (ADM)
  - Anomaly detection
  - Lost sources
- Data-driven methods based on
  - Machine learning (RNNs)
  - Maximum likelihood
  - Bayesian inference

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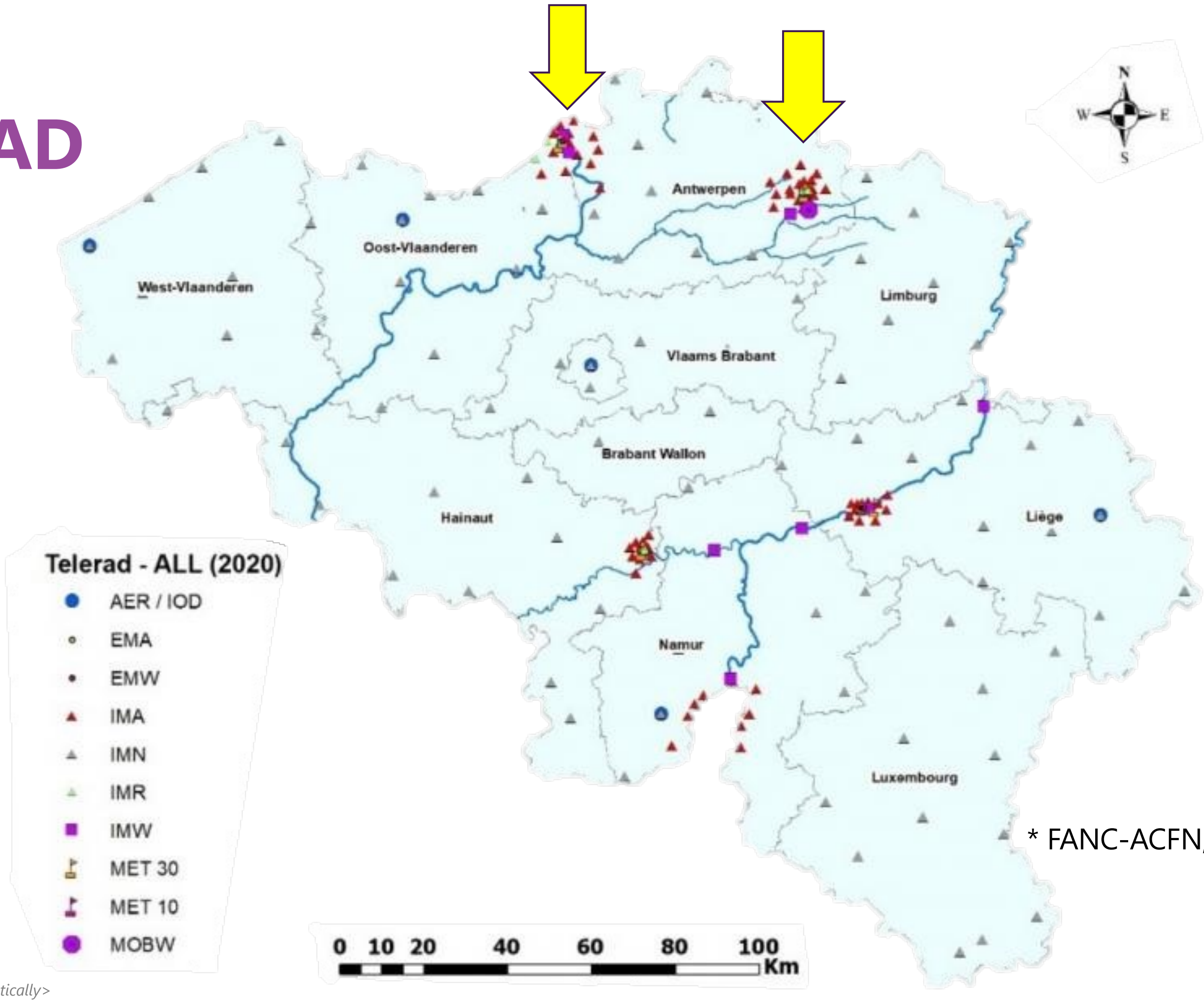

* FANC-ACFN, fanc.gov.be

Bayesian inference cookbook

$$f_{M|D}(m|d) = \frac{f_{D|M}(d|m) f_M(m)}{f_D(d)}$$

1. Define likelihood $f(d|m)$
2. Define prior $f(m)$
3. Sample posterior $f(d|m)$
4. Predict

1. Defining the likelihood

- Let there be a dense network of k detectors that measure N time intervals

* Liu et al (2018), 10.1371/journal.pone.0205092

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- In a Bayesian world, all observations are distributed as the joint pdf

$$f(D_{11}, \dots, D_{k1}, \dots, D_{1N}, \dots, D_{kN})$$

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$$f(D_1, \dots, D_k) \sim \mathcal{N}_k(\boldsymbol{\mu} = \boldsymbol{S}; \boldsymbol{\Sigma} = \mathrm{diag}[\boldsymbol{\sigma}]\, \boldsymbol{\rho}\, \mathrm{diag}[\boldsymbol{\sigma}])$$

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- So the likelihood is a multivariate normal

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2. Defining the prior

- Prior on the time-independent background

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- Prior on the scale vector (size of noise)

$$\boldsymbol{\rho} \sim \mathrm{LKJDistribution}(\eta = 1)$$

3. Sample the posterior

- **PyMC**: probabilistic programming library for Python
  - Easy model construction
  - State-of-art MCMC solvers (default: NUTS)
- Computational optimization using **PyTensor**
- Convergency tests and post-processing via **ArviZ**

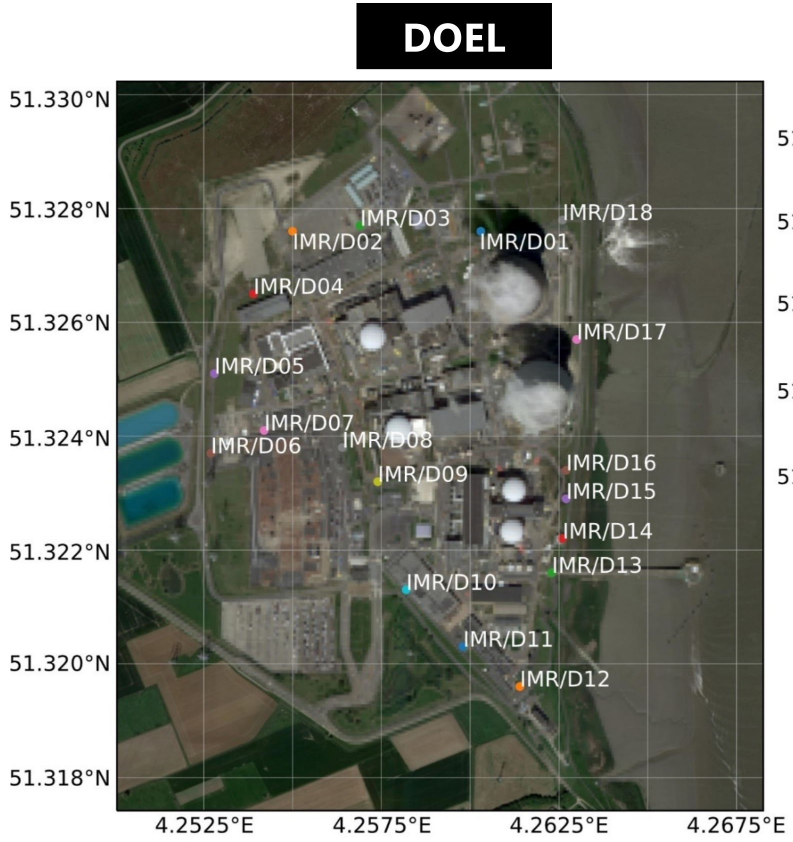
4. Verify and predict

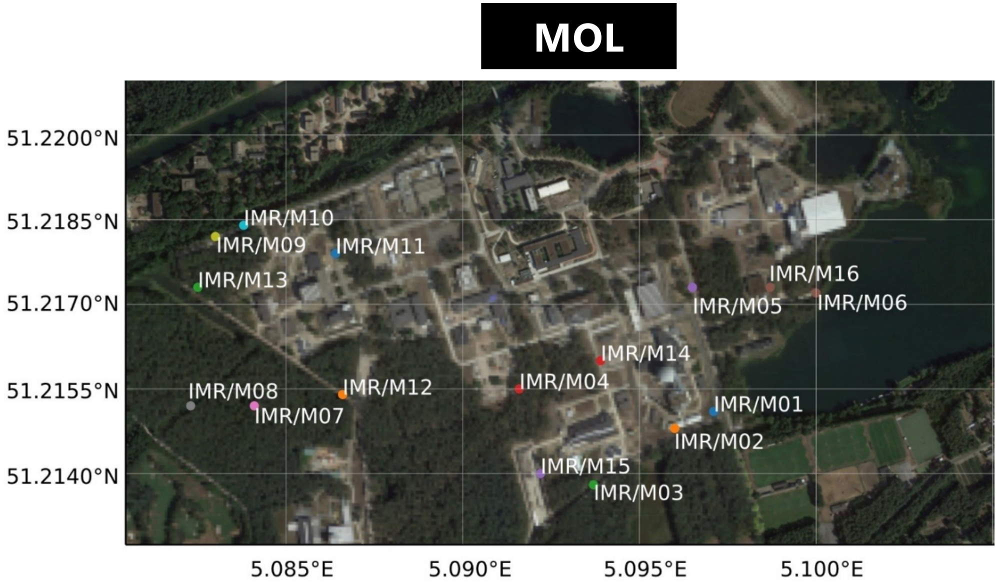
- Verification
  - Convergency checks
  - Posterior predictive

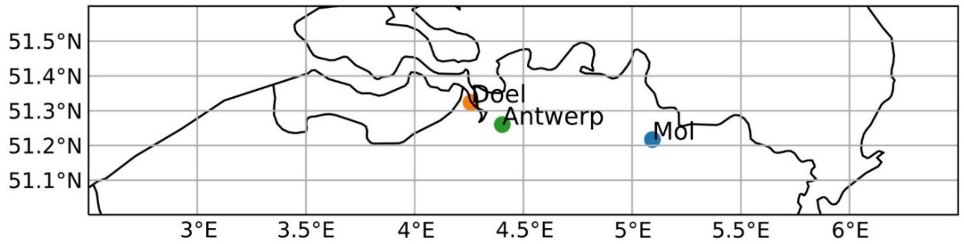
- Prediction
  - Conditional distribution

analytical for multivariate normal!

Can we put the mathematics into practice?







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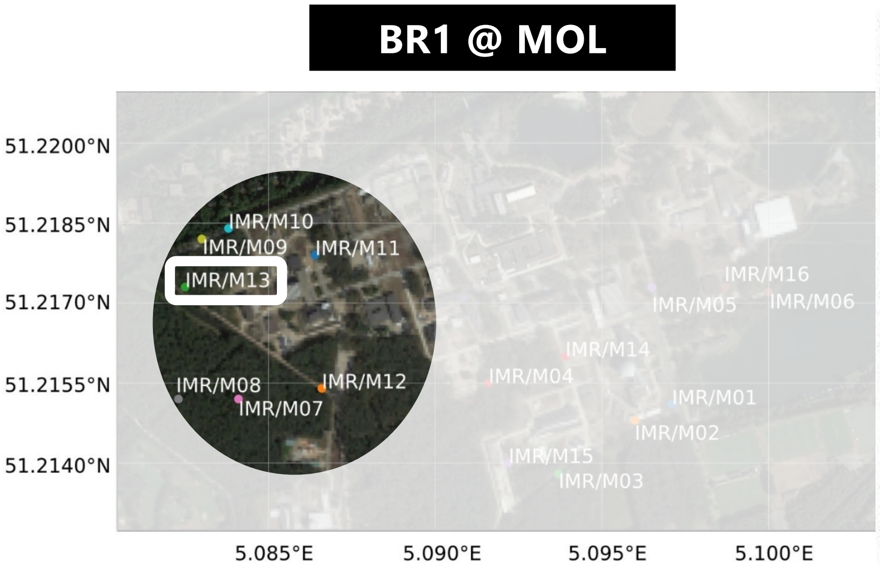
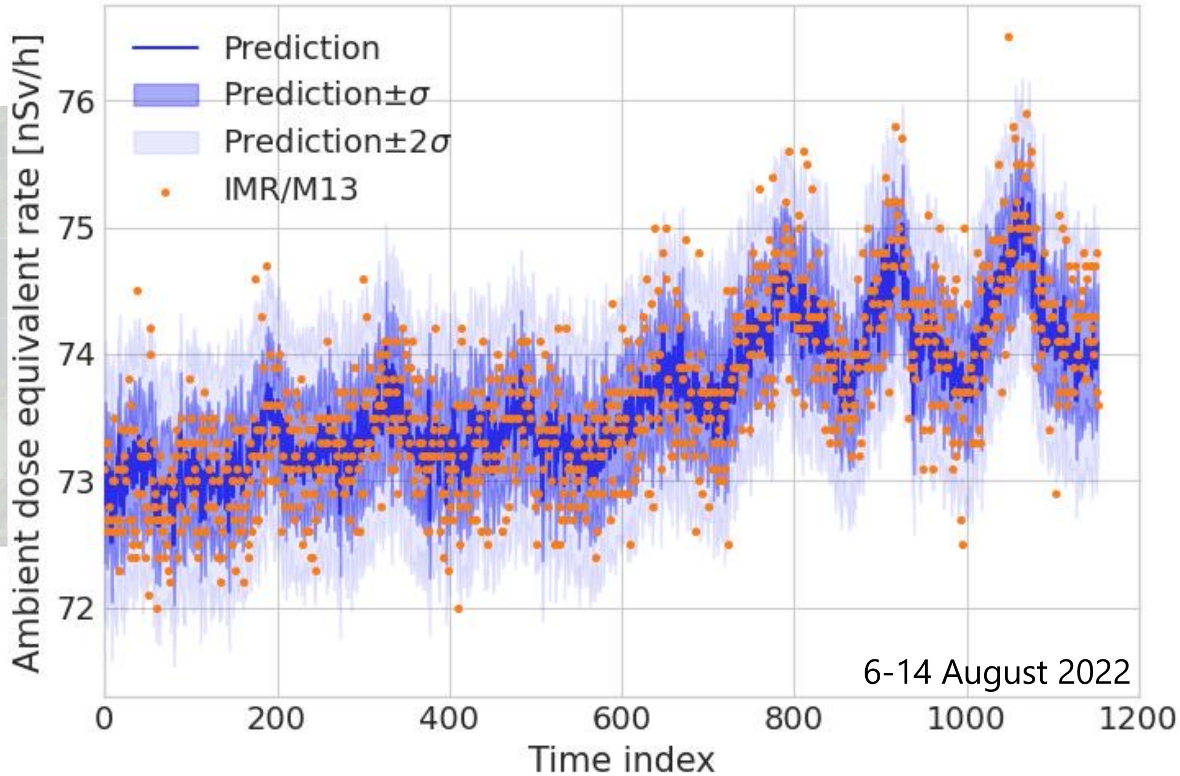
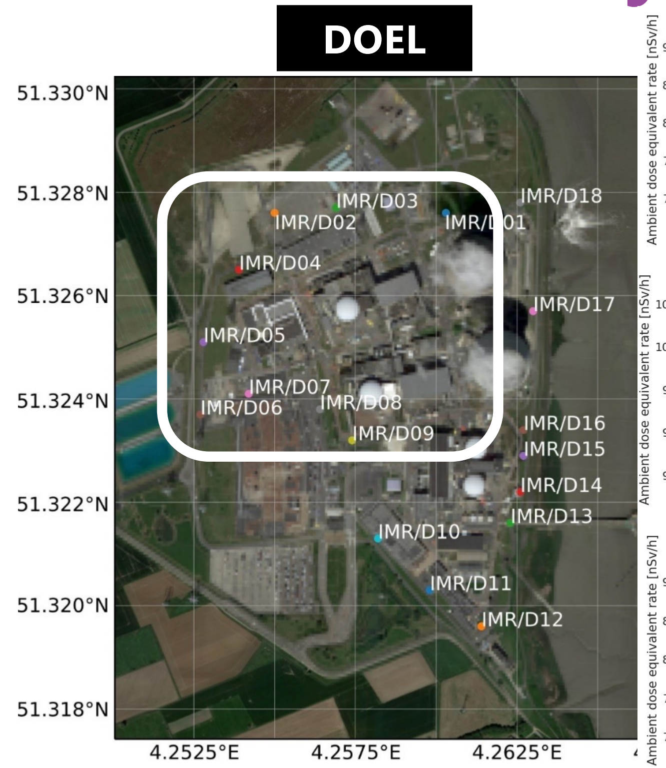
ISC: Restricted

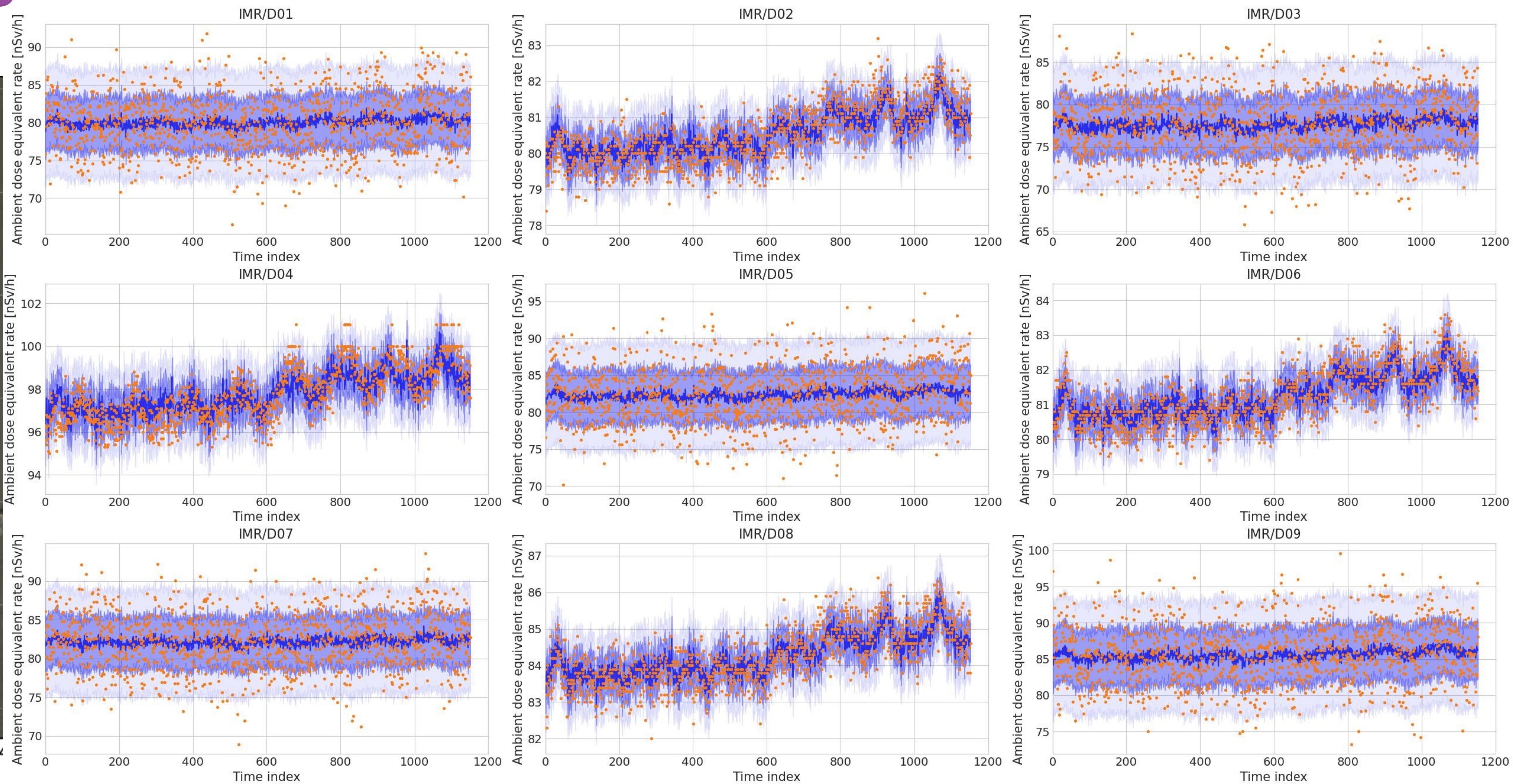
The calibrated model matches the training data very nicely…



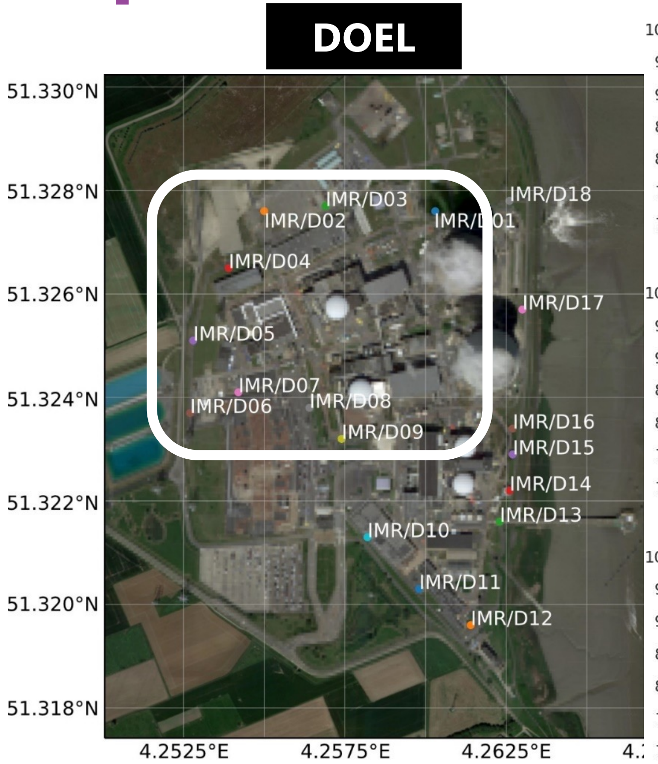


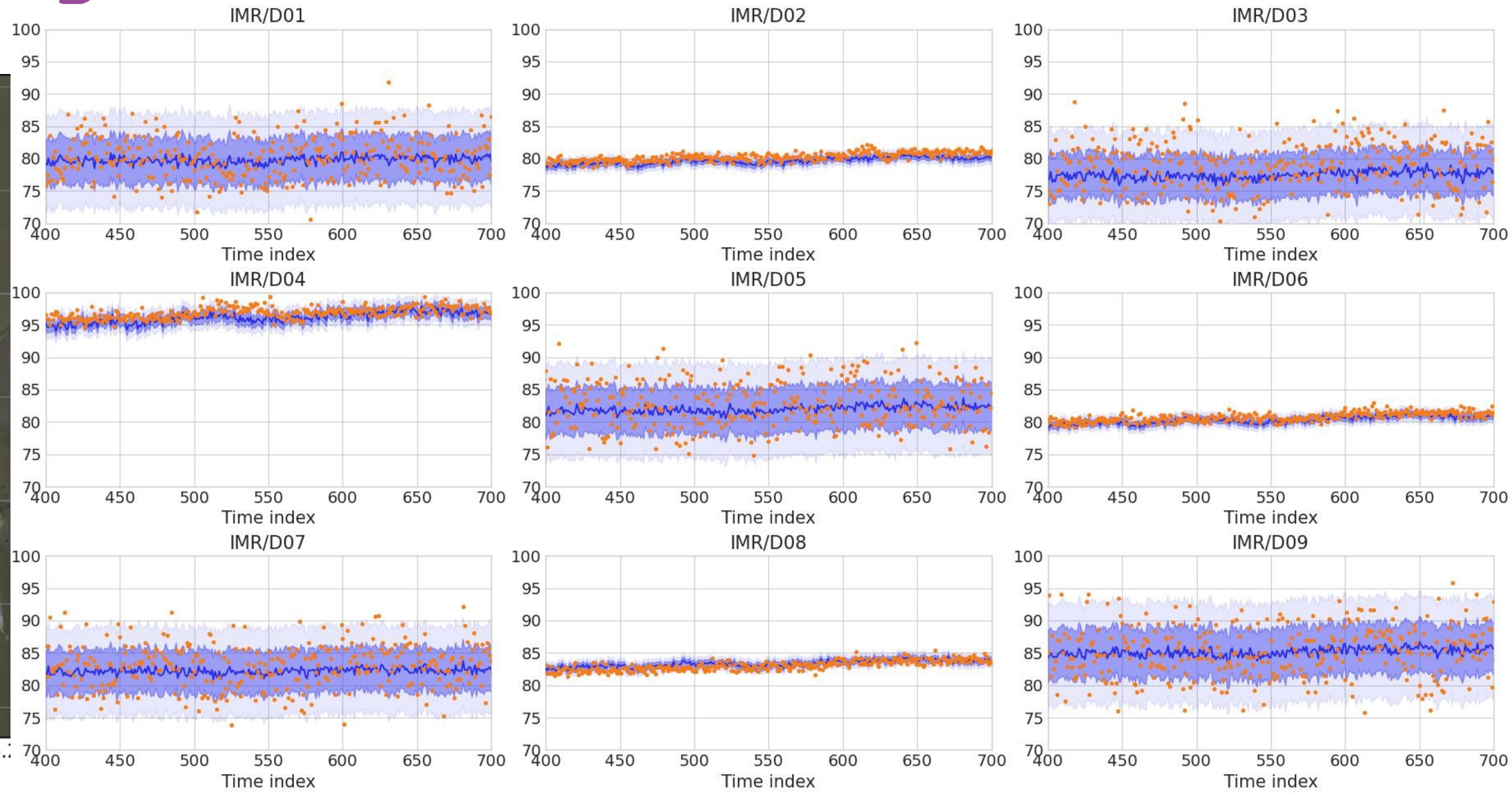
...and the correspondence is not limited to one site. It works just as well for the Doel site...



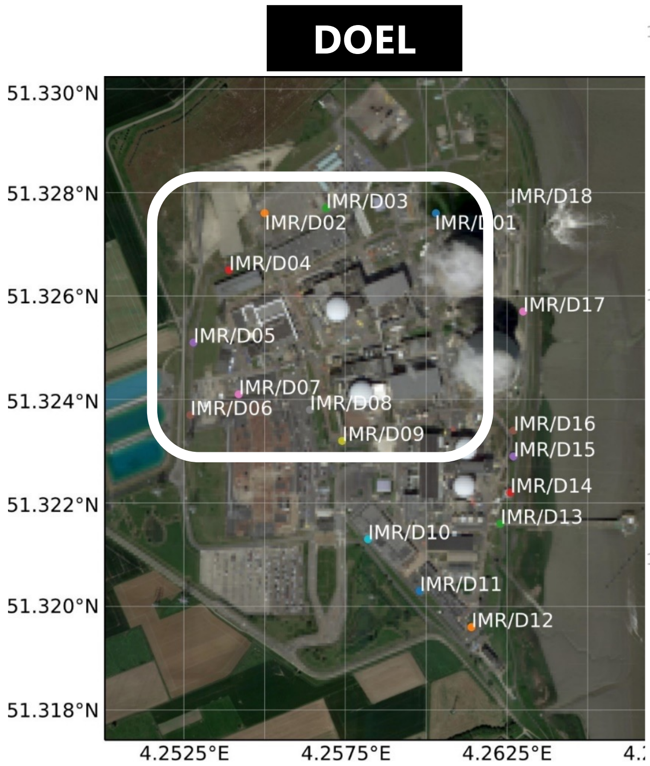


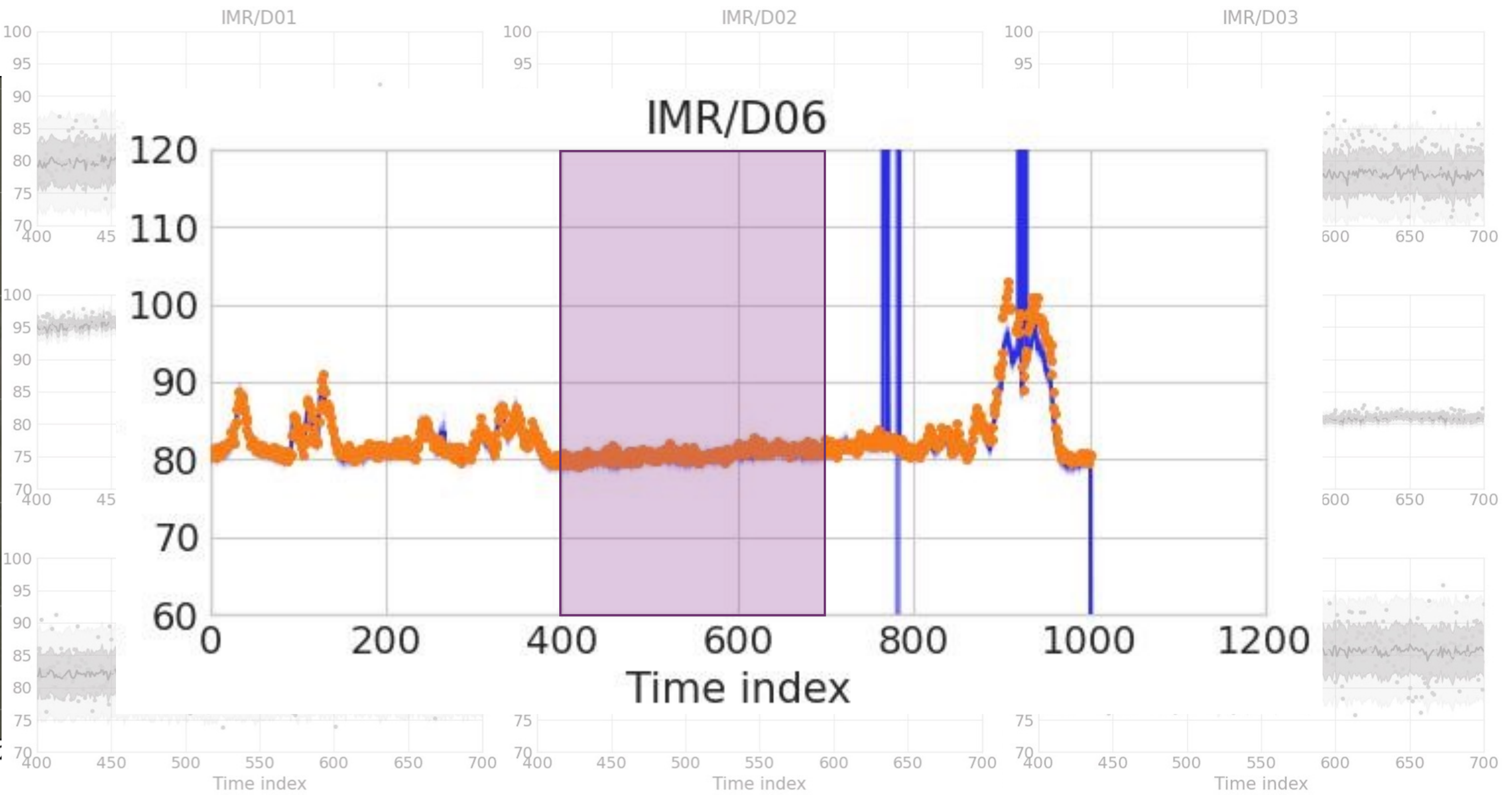
...and the calibration is stable in time. This is a prediction using data from 4 weeks later!



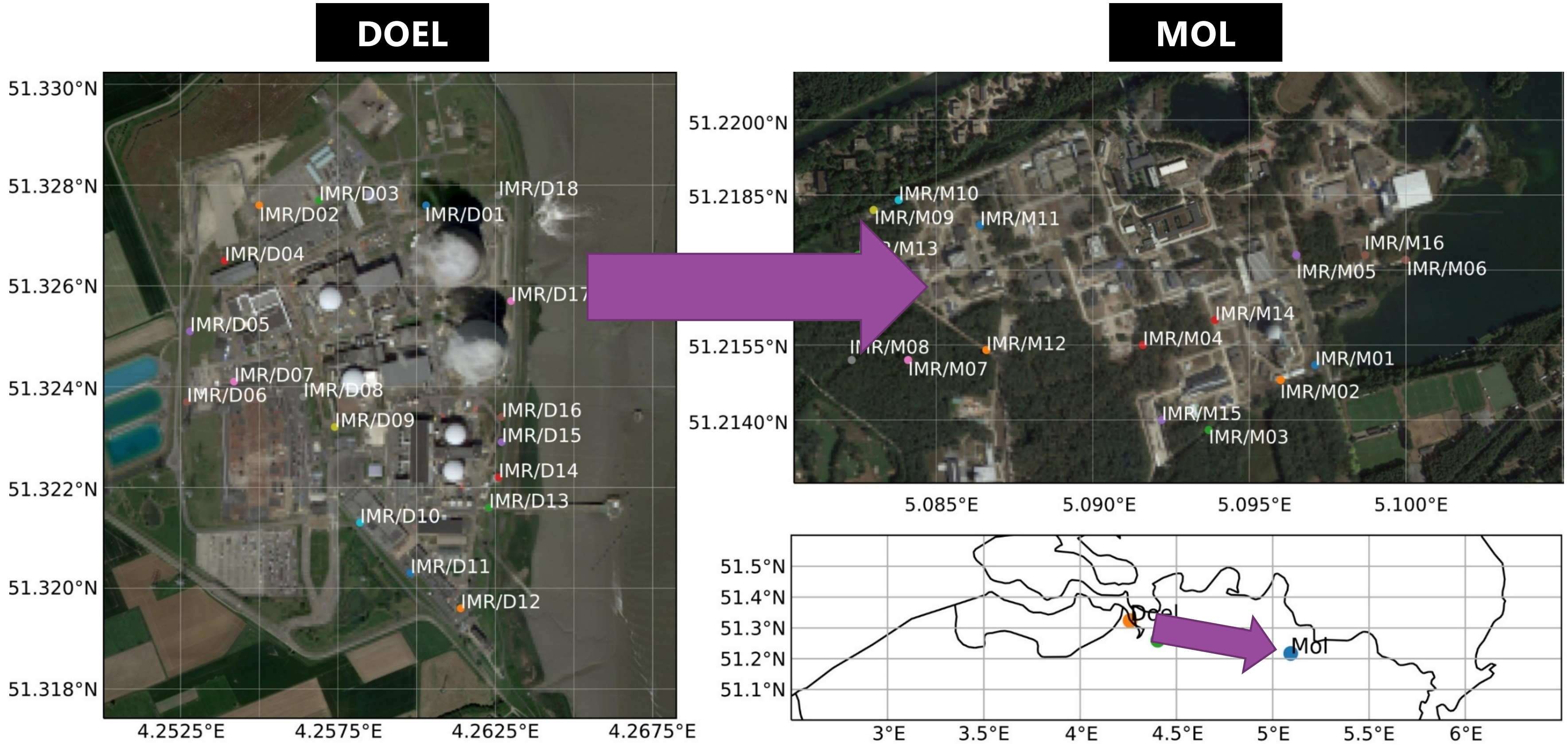


Rain peaks are also predicted rather well





Beyond dense networks: Doel predictive for Mol?



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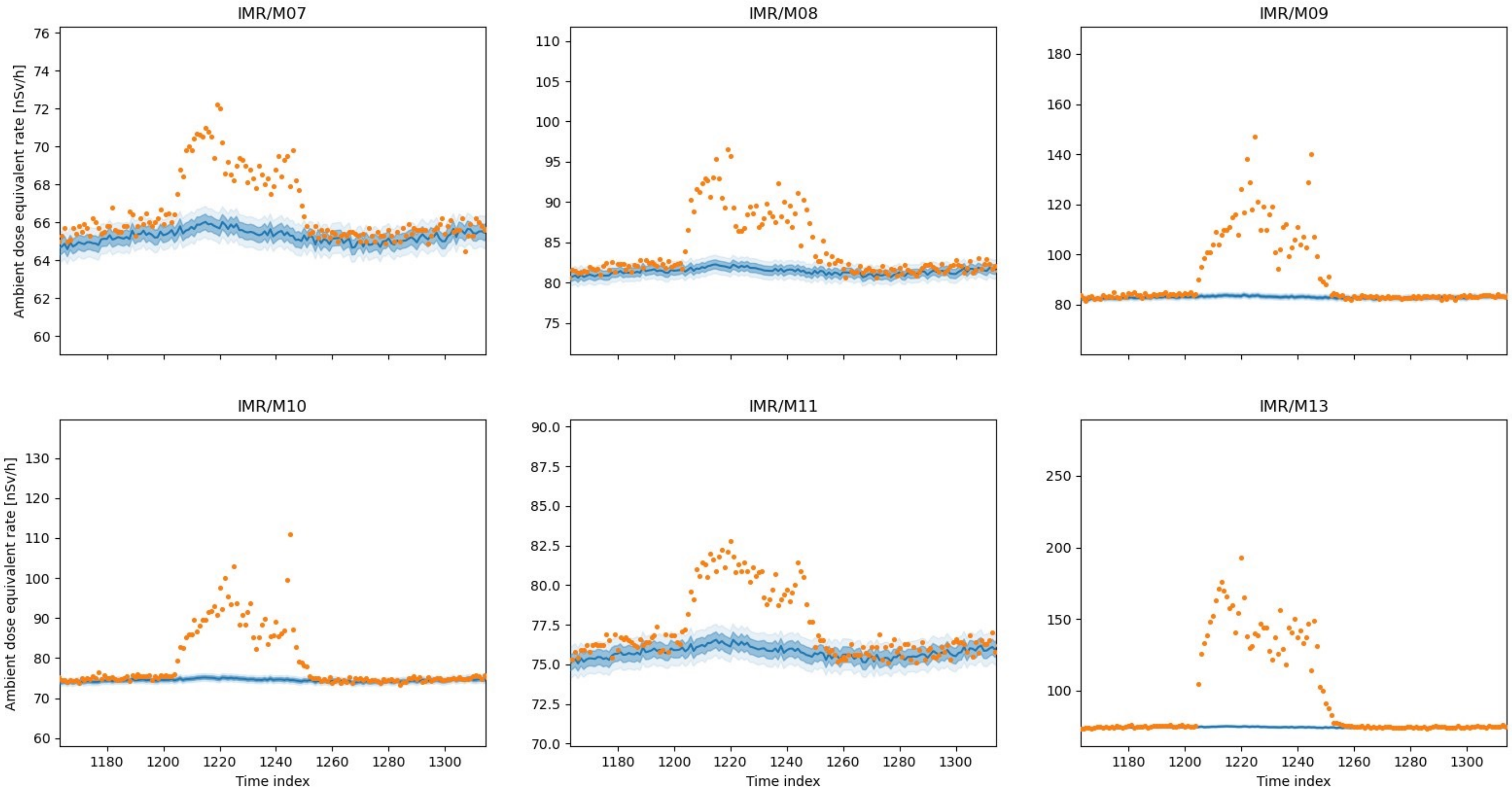

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Observations

$$Q_{o,i} = \frac{Observation(i) - Background\ estimate}{Dispersion\ model}$$

Likelihood

$$f(\boldsymbol{Q}_o|Q_a) = (2\pi)^{-\frac{k}{2}} \prod_{i=1}^{k} \frac{1}{\sigma Q_{\mathrm{o},i}} \exp\left[-\frac{1}{2} \frac{\ln[Q_{\mathrm{o},i}/Q_{\mathrm{a}}] - \mu}{\sigma^2}\right]$$

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Prior Q_a

$$Q_a \sim \text{Exp}(\lambda) \text{ with } \lambda = \frac{1}{16 \text{ MBq/s}}$$

Prior σ

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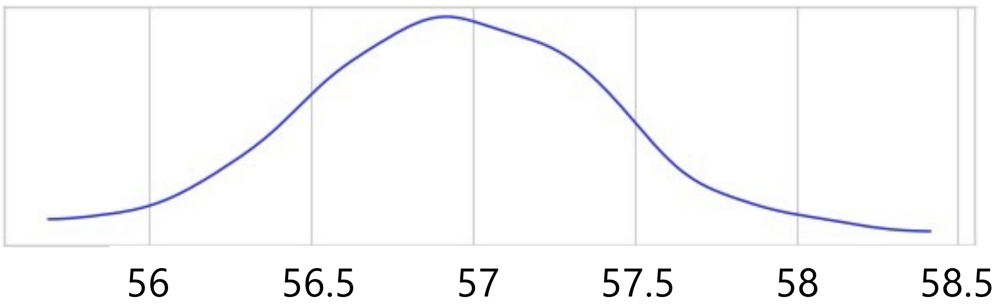
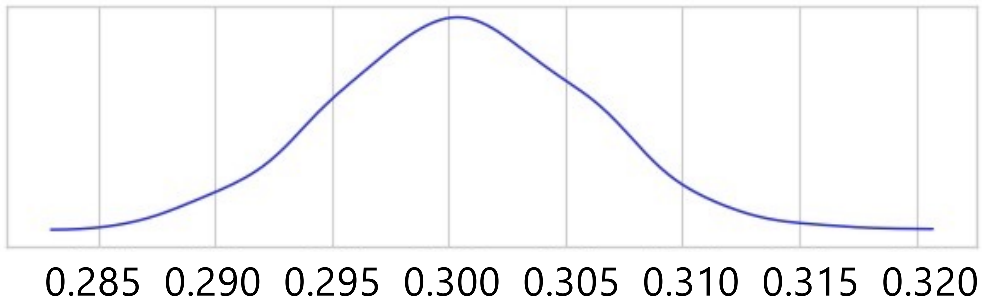
Posterior estimates of the source term and model error based on ADDER* model & Telerad observ.

Source term Q_a (MBq/s)

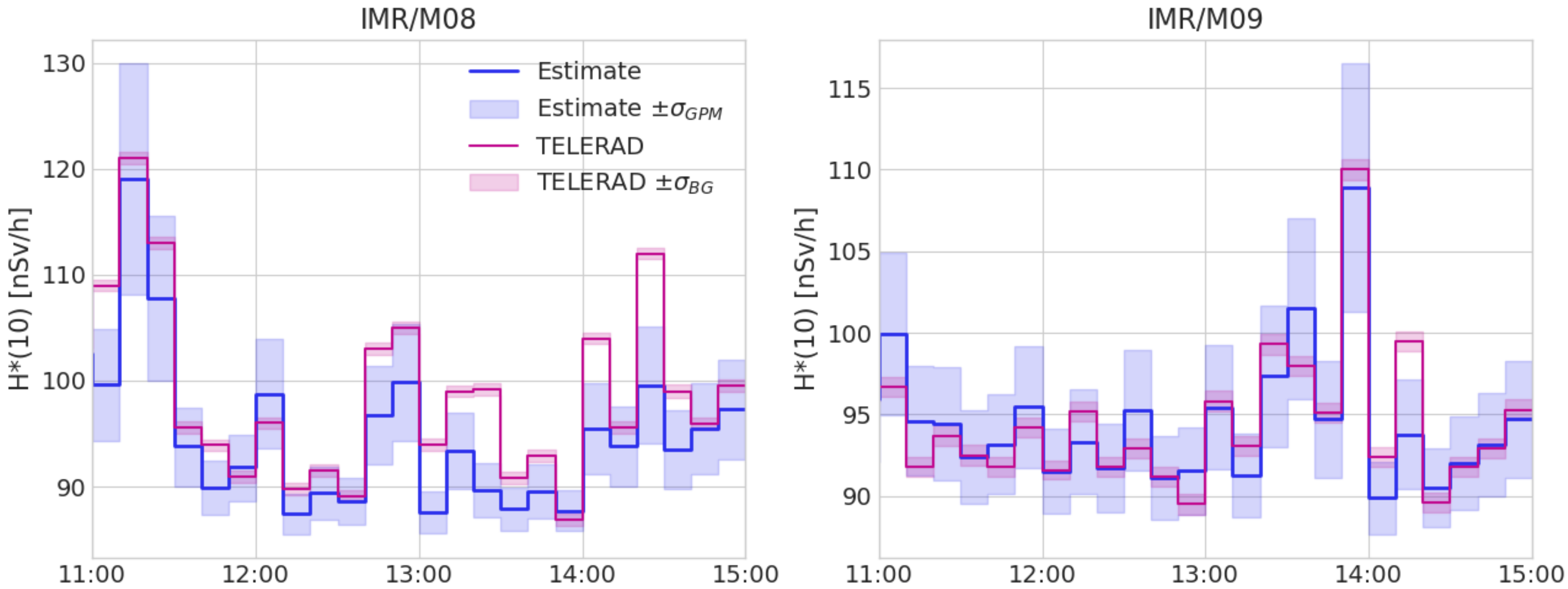


Model error σ



* Frankemölle et al (2022), 10.1016/j.jenvrad.2022.107012

Dose prediction using calibrated model versus actual Telerad observations



Discussion and outlook

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- Background model works very well despite strong simplifying assumptions
- Derivation from Bayes's theorem makes these assumptions explicit
- Doel dose rate is a good predictor for Mol despite distance (60km)

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Outlook on dispersion (work in progress)

- Formulate model error that includes spatial decorrelation (eddies)
- Go beyond just source inversion by updating multiple parameters (e.g., wind speed) → model calibration